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This report concerns several aspects of damping and control in distributed parameter systems, with emphasis on applications to elastic and viscoelastic structures. For viscoelastic bodies, the ineffectiveness of boundary feedback for damping nonoscillatory "creep" decay was demonstrated, and a precise energy space formulation was developed for the viscoelastic wave equation. In addition, a reachability result was proved for a second-order linear integral equation. For elastic beams and plates, exact controllability and the exponential decay of energy were established in new settings. These include uniform exponential decay of energy by means of locally distributed damping in a one-dimensional nonhomogeneous medium, and, under certain boundary conditions, for longitudinal vibrations in a thermoelastic rod.

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FINAL TECHNICAL REPORT

ENERGY DECAY AND BOUNDARY CONTROL FOR DISTRIBUTED PARAMETER SYSTEMS WITH VISCOELASTIC DAMPING

AFOSR GRANT AFOSR-89-0268

for the period

15 May 1989 - 14 November 1990

by

K. B. Hannsgen and R. L. Wheeler

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I. SUMMARY

The subject grant supported research on damping and control in distributed systems, with emphasis on applications to elastic and viscoelastic structures. For viscoelastic bodies, the ineffectiveness of boundary feedback for damping nonoscillatory "creep" decay was demonstrated, and a precise energy space formulation was developed for the viscoelastic wave equation. In addition, a reachability result was proved for a second-order linear integral equation. For elastic beams and plates, exact controllability and the exponential decay of energy were established in new settings.

II. INTRODUCTION

This report contains a summary of the results partially supported under the Air Force Office of Scientific Research Grant AFOSR-89-0268 during the period 15 May 1989 to 14 November 1990. This project was concerned with the stabilization and control of elastic and viscoelastic structures. In particular, the interaction between active and passive internal damping mechanisms is a recurring issue in the work. The principal investigators were Professors Kenneth B. Hannsgen and Robert L. Wheeler.

During the eighteen month period, eleven research papers were completed. Also, four faculty (two principal investigators, one senior investigator, one postdoc) and five graduate students were partially supported by the grant; these students are expected to finish their degree work (4 Ph. D., 1 M. S.) in the near future.

A number of specific topics were studied in the research and highlights are described below. Accomplishments included

1.) a demonstration that boundary feedback is ineffective, and can even be counterproductive in stabiliziing the motion of a viscoelastic body [1,2],

2.) development of a representation formula for solutions of the viscoelastic wave equation with boundary feedback damping in terms of solutions for the free and fixed end cases, with consequent sharp results on energy decay rates [3],

3.) controllability (or reachability) results for an aeroelastic plate, for an Euler-Bernoulli equation with variable coefficients, for a second-order integrodifferential equation, and for a general class of abstract parabolic systems [4, 8, 10, 11],

4.) a proof of uniform exponential decay of energy for a one-dimensional nonhomogeneous medium with locally distributed damping and, under certain boundary conditions, for longitudinal vibrations in a thermoelastic rod [5, 7].

III. HIGHLIGHTS OF RESEARCH

In this section we discuss some of the work performed under the grant. Reprints of the papers will be forwarded to AFOSR as soon as they become available.

(1) Damping of motion in viscoelastic bodies.

The motion of a body with linear viscoelastic damping can sometimes be modelled by an integrodifferential equation

$$\mathbf{u}' = A * \mathbf{L}\mathbf{u} \tag{1}$$

in a Hilbert space. Here A is a scalar memory kernel and, for suitable homogeneous boundary conditions, $\mathbf{L}$ is a self-adjoint negative definite linear operator. $*$ denotes convolution on $[0, \infty)$. A is positive, decreasing and convex with a positive limit E at infinity. It is known that if $A(t) - E$ decays exponentially, then so does $\mathbf{u}(t)$; in joint work with Yuriko Renardy (1987) Hannsgen and Wheeler showed, via characteristic root analysis in the frequency domain, that this decay rate can be improved by the application of a boundary feedback force. In [1, 2] an analysis of (1) in a weighted L^1 space was used to determine precise decay rates for the energy in the cases without boundary feedback. For cases where $A(t) - E$ decays only algebraically, the new result gave sharp information on the (algebraic) decay rate of $\mathbf{u}(t)$. By an indirect argument it was then shown that boundary feedback damping does not improve the decay rate (and can even lead to slower decay) if that rate is not exponential or, more generally, if nonoscillatory (creep) motion dominates all the vibrating modes for large time.

In [3] the study of decay rates in energy spaces was extended (for the one-dimensional viscoelastic wave equation only) to the case where boundary feedback is applied. It was shown that the solution can be represented as a relatively simple combination (a frequency dependent superposition) of solutions of two problems of the form (1) with self-adjoint $\mathbf{L}_1$ and $\mathbf{L}_2$. This leads to precise well-posedness results and estimates of energy decay rates.

(2) Controllability.

The results in this area involve several general approaches: solvability of moment problems [4], the Hilbert uniqueness method and unique continuation techniques [8, 10, 11]. In addition, the special properties of particular systems, reflecting the underlying physics of the mechanical structures being modelled, require individual treatment.

In [4], a structurally damped Euler-Bernoulli plate is analyzed as a representative for a class of abstract parabolic problems for which exact null controllability in arbitrary time can be established through moment-problem methods.

The paper [8] also concerns exact controllability of a plate modelled by the Euler-Bernoulli constitutive assumption, but with the additional consideration of self-induced aerodynamic pressure. Exact internal controllability is proved in the one-dimensional case for both subsonic and supersonic motion.

In [10], the Euler-Bernoulli model is considered again, but this time the dimension is arbitrary and spatial dependence is considered both in the coefficients of the basic equation and in the restriction of the distributed controlling forcing term to a neighborhood of the boundary. Under these conditions, exact controllability in arbitrary time is established.

Finally, in [11], reachability is established for a class of second-order integrodifferential equations. The paper is significant for showing how unique continuation arguments can be used to obtain this kind of result even when the integral kernel is not assumed to be small.

(3) Exponential decay of energy.

Exponential decay rates provide one important measure of damping effectiveness. The damping effect of heat dissipation on mechanical vibrations leads to fairly subtle mathematics.

In [5], the case of longitudinal vibrations in a thermoelastic rod is considered, and uniform exponential decay of energy is established under certain boundary conditions: Dirichlet and Neumann conditions, respectively, on the displacement and temperature, or the reverse.

(Very recently, and by entirely different methods, Kim has shown exponential decay for the case of Dirichlet conditions on both unknowns.)

Article [7] concerns exponential decay of energy by means of locally distributed damping in a one-dimensional nonhomogeneous medium. Energy arguments based on wave propagation properties are used.

IV. CUMULATIVE LIST OF PUBLICATIONS

AFOSR Grant AFOSR-89-0268

1. K. B. Hannsgen and R. L. Wheeler, Viscoelastic and boundary feedback damping: precise energy decay rates when creep modes are dominant, J. Integral Equations and Applications, to appear.

2. K. B. Hannsgen and R. L. Wheeler, Moment conditions for a Volterra equation in a Banach space, Proc. Clarement Conf. on Differential Equations and Applications to Biology and Population Dynamics, to appear.

3. K. B. Hannsgen and R. L. Wheeler, A representation formula for one-dimensional viscoelasticity with stabilizing boundary feedback, submitted.

4. S. W. Hansen, Bounds on functions biorthogonal to sets of complex exponentials; control of damped elastic systems, J. Math Anal. Appl., to appear.

5. S. W. Hansen, Exponential energy decay in a thermoelastic rod, submitted.

6. S. W. Hansen and G. Weiss, The operator Carleson measure criterion for admissibility of control operators for diagonal semigroups on l^2, Systems and Control Letters, to appear.

7. J. U. Kim, Exponential decay of the energy of a one-dimensional nonhomogeneous medium, SIAM J. on Control and Optimization, to appear.

8. J. U. Kim, Exact internal controllability of a one-dimensional aeroelastic plate, Applied Math. and Optimization, to appear.

9. J. U. Kim, A unique continuation property of a beam equation with variable coefficients, Proc. International Conference on Control and Estimation of Distributed Parameter Systems, Vorau, Austria, 1990, to appear.

10. J. U. Kim, Exact semi-internal control of an Euler-Bernoulli equation with time-dependent coefficients, submitted.

11. J. U. Kim, Control of a second order integro-differential equation, submitted.

V. INVESTIGATORS PARTIALLY SUPPORTED UNDER THIS GRANT

K. B. Hannsgen - Principal Investigator

R. L. Wheeler - Principal Investigator

J. U. Kim - Senior Investigator

Scott W. Hansen - Postdoctoral Research Associate

Scott Inch - Graduate Student (Working on Ph.D.)

Hamadi Marrekchi - Graduate Student (Working on Ph.D.)

Rubin Spies - Graduate Student (Working on Ph.D.)

Moshen Tadi - Graduate Student (Working on Ph.D.)

Kimberly Oates - Graduate Student (Working on Masters)

VI. INTERACTIONS

Presentations at Professional Meetings and Other Universities

Hannsgen

1. "Stabilization and energy decay in viscoelastic structures" at SIAM Conference on Control in the 90's, San Francisco, California, May, 1989, (contributed talk).

2. "Moment conditions for a Volterra integral equation in a Banach space" at Claremont Conference on Differential Equations and Applications to Biology and Population Dynamics, Claremont, California, January, 1990.

3. "Energy decay rates for systems with viscoelastic damping", Colloquium talk at Georgetown University, Washington, DC, March, 1990.

Wheeler

1. "Feedback stabilization in systems with viscoelastic damping" at SIAM Conference on Control in the 90's, San Francisco, California, May, 1989, (invited minisymposium talk).

2. "Stabilization of integro-partial differential equations with slowly decaying kernels" at International Conference on Differential Equations and Applications, Retzhof, Austria, June, 1989.

3. "Viscoelastic and boundary feedback damping: precise energy decay rates when creep modes are dominant" at Special Session on Control of Infinite Dimensional Systems, 96th Annual Meeting of Amer. Math. Soc., Louisville, Kentucky, January, 1990, (invited talk).

4. "Internal dissipation and boundary feedback dissipation in one-dimensional viscoelasticity" at International Conference on Control and Estimation of Distributed Parameter Systems, Vorau, Austria, July, 1990.

Kim

1. "Approximation of the motion of a Bingham fluid" at Special Session on Mathematical Fluid Dynamics, 851st Meeting of Amer. Math. Soc., Hoboken, New Jersey, October, 1989, (invited talk).

2. "Control of an aeroelastic plate" at SIAM Conference on Dynamical Systems, Orlando, Florida, May, 1990, (contributed talk).

3. "Internal exact controllability of an Euler-Bernoulli equation" at International Conference on Control and Estimation of Distributed Parameter Systems, Vorau, Austria, July, 1990.

4. "Energy dissipation and exact controllability", in Partial Differential Equations Seminar at University of California at Berkeley, Berkeley, California, November, 1990.

Hansen

1. "Thermal boundary control of a one-dimensional linear thermoelastic rod", at SIAM Annual Meeting, Chicago, Illinois, July, 1990.

In addition to the above presentations by the personnel associated with this grant, Wheeler, together with David L. Russell, co-organized a minisymposium " New Methods in Control of Distributed Parameter Systems" for the SIAM Annual Meeting held in Chicago, July 16-20, 1990. The speakers were Goong Chen, Vilmos Komornik, John Lagnese and Günter Leugering.